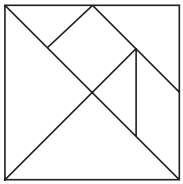
INTRODUCTION

A Tangram is an ancient puzzle, invented in China and consisting of a square divided into seven geometric shapes:

- Two large right triangles
- One medium right triangle
- Two small right triangles
- One square
- One parallelogram



Tangram

The shapes' areas are related in the following ways:

- The large triangle is twice as large as the medium triangle.
- The large triangle is twice as large as the parallelogram.
- The large triangle is twice as large as the square.
- The large triangle is four times as large as the small triangle.

Children should be encouraged to work cooperatively in pairs or small groups to explore the pieces and verbalize their discoveries about colors, shapes, sizes, and relationships.

These activities engage children in problems and challenges that will help them gain a deeper understanding of geometric concepts. As they manipulate the tangram pieces, students will improve visual thinking and spatial sense. Tangrams also have a great value in the hands-on teaching of geometric shapes, congruence, similarity, symmetry, area, and beginning transformational geometry.

LEARNING ABOUT TANGRAM SHAPES

Distribute a set of Tangrams to each student or cooperative group. Initiate a discussion about the pieces. Talk about similarities and differences among the pieces. Students may notice that some of the pieces are the same size and shape, while others are the same shape but larger or smaller. Some shapes will have more "corners" than others.

Ask children to trace each of the pieces onto paper and then color them. Have them then exchange papers and practice covering the traced shapes with the actual tangram pieces.

Be sure each student has a set of Tangrams. Display a single tangram piece on the overhead projector or draw the shape on the board. Discuss the characteristics of the piece. Ask students to hold up a piece that matches the one being displayed. As students become familiar with the pieces, play the game by simply describing the piece rather than displaying it.

The piece I want has four sides and four corners. **[the square]**

Younger children will use common language to name or describe tangram pieces. Gradually insist that students begin to use mathematical language as they become more acquainted with geometric shapes.

SLIDES, TURNS, AND FLIPS

Activity 1

On the overhead projector, demonstrate a slide incorporating the directions up, down, left, and right. Show a diagonal slide. Have students place one of the large triangles in the center of their desk. Have them slide the piece following your verbal directions:

- Slide the piece up to the right corner of the desk.
- Slide the piece down to the edge of the desk.
- Slide the piece diagonally to the lower left corner of the desk.

Encourage students to discover that no matter which direction a piece slides, the shape remains unchanged.

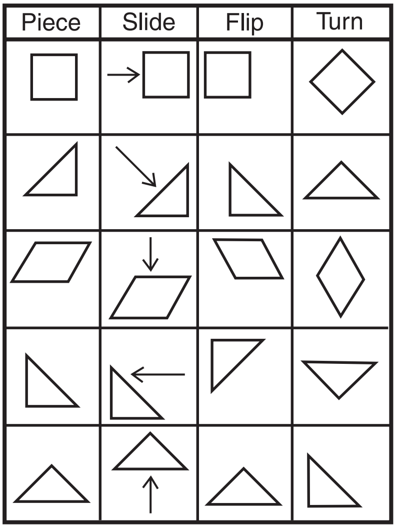
Introduce the concept of turns on the overhead by rotating one of the largest triangles through 360°. Incorporate correct terminology, such as "quarter turns" and "90° rotation." Demonstrate that rotations can be effected in either a clockwise or counterclockwise direction. Point out that the shape remains the same no matter how many degrees the rotation.

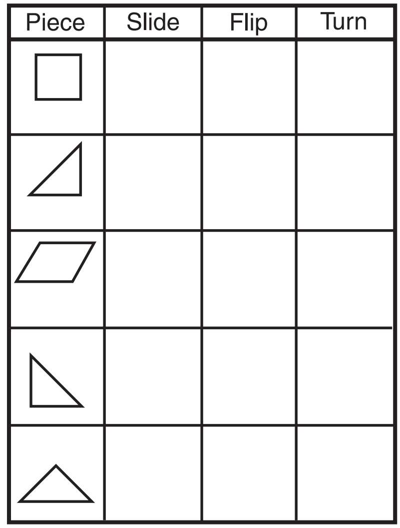
Give each pair of students 10 tangram triangles all of the same size and a sheet of newsprint. Ask the students to place a dot at the approximate center of paper. Using one of the 45° angles, at the dot, demonstrate how to arrange the triangles, petal-like, rotated around the point. After each group has successfully placed the triangles around the point, ask the students to carefully trace their pieces in position. Have younger students color the drawing, adding a stem and leaves to make a flower. Older children should be challenged to place the 10 triangles equidistant around the circumference of a circle, using a compass and a protractor to discover the placement interval of 36°. Their drawings should include the measurement work done. Display the results.

Demonstrate a flip to the class on the overhead and ask them to follow along using their sets of Tangrams. Encourage exploration and discussion that will bring about the discovery that when a piece is flipped, it becomes a mirror-image (or reflection) of the unflipped piece. Challenge students to find those pieces for which a flip is the same as a turn.

Activity 2

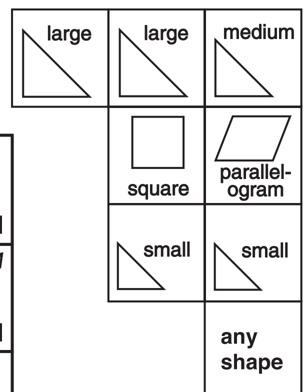
Have students work in groups of three. Draw the prototype chart on the next page. Ask each team to duplicate the chart on a sheet of paper. Then, using a set of Tangrams, have each team experiment and manipulate the pieces to discover positions for each piece which will complete the chart. The piece should be traced in position in the correct box on the chart. Advise students that there are many correct answers. When all teams are finished, compare charts and discuss results.

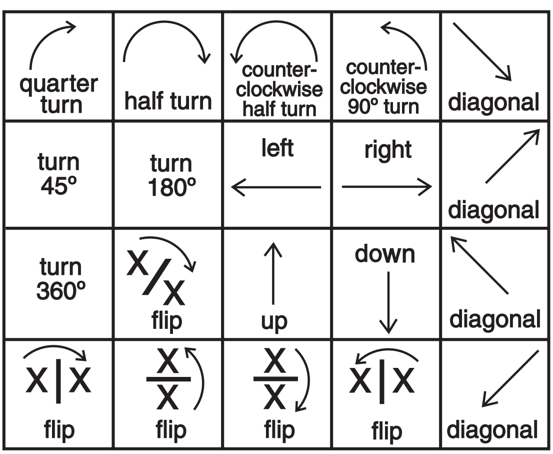




Activity 3

Cut tagboard into 28 two-inch squares and draw 2 decks of cards that look like this:





Shuffle each deck and place the 2 decks face down on the table. Put a pile of tangram pieces in the center of the table. The first student draws the top card from each deck, chooses the piece indicated, and performs the activity indicated on the card. The student scores 2 points for completing the task correctly. Continue around the table, taking turns drawing cards and following the directions. The first student to reach 10 points wins the game.

Activity 4
Give each pair of students two small triangles. Have one student put the pieces together to form a shape and then trace around it. Have the partner make a new shape by sliding, turning, or flipping both triangles together. Trace the new shape. Switch roles and repeat.

COVERING SHAPES

Give each pair of students two sets of Tangrams. Instruct students to put the square in front of themselves.

- Which two pieces will exactly cover the square? **[2 small triangles]**
- Examine the parallelogram and determine which two pieces cover it exactly. **[2 small triangles]**
- Which two pieces will cover the medium triangle? **[2 small triangles]**

Tell the students that the large triangle can be covered with four different combinations of other pieces.

Use your tangrams to find the four ways and record your solutions by tracing the pieces on sheets of paper. **[2 medium triangles; medium triangle and 2 small triangles; square and 2 small triangles; parallelogram and 2 small triangles]**

Have students work in pairs. Ask each group to use any two tangram pieces to form a shape. Trace the shape outline on a piece of paper and remove the pieces. Have groups exchange shape outlines and attempt to fill the puzzle with the correct pieces. As students progress, this activity can be extended to using three or more pieces to build the shape.

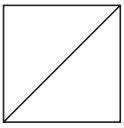
MAKING GEOMETRIC SHAPES

Since there are several solutions to many of the problems in this section, encourage students to think about all the possible solutions. Then have them work cooperatively to record as many as they find.

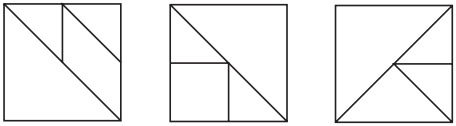
Give each student a complete set of Tangram pieces. Discuss the characteristics of a square: four sides; four right angles; sides of equal length; opposite sides parallel.

Draw a square that is $2\frac{3}{4}$" x $2\frac{3}{4}$". (Copies should be provided for younger children.)

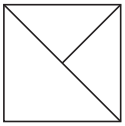
Cover the square using 2 tangram pieces from the set. Record your solutions.



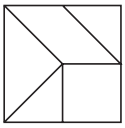
Cover the square using 4 tangram pieces. There are three possible combinations of pieces which will successfully cover the square. Find and record them.



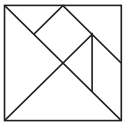
Can a square be made using just 3 pieces? **[yes]**



Using 5 pieces? **[yes]**



Using 7 pieces? **[yes]**



Using 6 pieces? **[no]**

Draw a right triangle with side lengths of 4". (Copies should be provided for younger children.)

Cover the shape using 2 tangram pieces.

[Use two large triangles.]

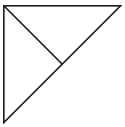
Cover the triangle with 4 tangram pieces and complete a chart to show your solutions.

Compare and discuss student findings.

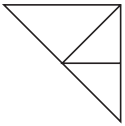
Is there more that one solution? **[yes, three]**

Covering a Right Triangle with Tangrams			
Tangram Piece	**Quantity**	**Quantity**	**Quantity**
Large △			
Medium △			
Small △			
□			
▱			

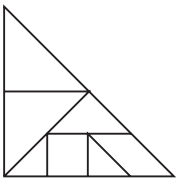
Can a triangle be made using 2 pieces? **[yes]** Record your solutions.



Using 3 pieces? **[yes]**



Using 7 pieces? **[yes]**



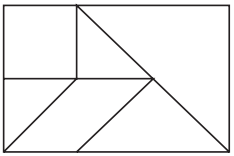
Discuss a rectangle and how it differs from a square.

Build a rectangle using 3 tangram pieces and chart your answers. **[medium triangle and 2 small triangles; parallelogram and 2 small triangles; square and 2 small triangles]**

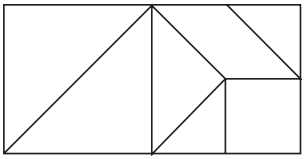
Build a rectangle with 4 pieces. **[square, 2 small triangles, and parallelogram; square, 2 small triangles, and medium triangle]**

Build a rectangle with 5 pieces. **[2 large triangles, medium triangle, and 2 small triangles; 2 large triangles, parallelogram, and 2 small triangles; 2 large triangles, square, and 2 small triangles]**

Build a rectangle with 6 pieces. **[large triangle, medium triangle, parallelogram, square, and 2 small triangles]**

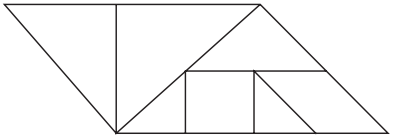


Build a rectangle using all 7 tangram pieces.



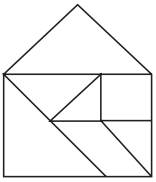
List the attributes of a parallelogram. Discuss similarities and differences between squares, rectangles, and parallelograms. Have the class explore building parallelograms using various numbers of tangram pieces.

Construct a parallelogram using all 7 tangram pieces.

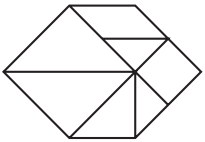


Initiate a discussion about pentagons and their characteristics.

Build a pentagon using all the tangram pieces.



Construct and trace a hexagon using all 7 tangram pieces.



SYMMETRY

Use an $8\frac{1}{2}$" x 11" sheet of paper to demonstrate the concepts of symmetry and lines of symmetry.

Fold the paper in half widthwise.

- Do the two sides match? **[yes]**

Then, refold in half lengthwise.

- Do the two sides match? **[yes]**

Finally, fold the paper in half along a diagonal.

- Do the two sides match? **[no]**

Explain that those fold lines which made the sides match are lines of symmetry for the sheet of paper.

Work in pairs. Ask students to trace each of the tangram shapes on a sheet of paper and carefully cut them out. On another sheet, have each team make a recording chart that looks like this:

lines of symmetry	-----	|	/	\
large triangle	[no]	[yes]	[no]	[no]
medium triangle	[no]	[yes]	[no]	[no]
small triangle	[no]	[yes]	[no]	[no]
square	[yes]	[yes]	[yes]	[yes]
parallelogram	[no]	[no]	[no]	[no]

One student on the team should fold each piece along the possible lines of symmetry. Students should together decide after each fold whether or not the shape is symmetrical across that particular fold line. The other student should record the team's findings by writing "yes" or "no" in the appropriate cell of the chart.

- Do the results change if any of the pieces are rotated or flipped? **[Yes, the lines of symmetry for all of the triangles change if the hypothenuse of the triangle is diagonal or vertical, rather than horizontal. Note that the number of lines of symmetry does not change, however.]**

Ask students to build a symmetrical shape using all 7 tangram pieces and show the lines of symmetry.

CONGRUENCE AND SIMILARITY

Introduce the terms and explain that **congruent shapes** have the same size and shape and that **similar shapes** have the same shape with congruent corresponding angles but are proportional in size.


congruent shapes


similar shapes

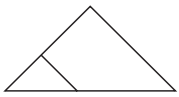

not similar

Work in pairs with two sets of Tangrams. One student displays a piece; the other counters with a congruent match. If the challenge is correctly met, the second student keeps the pair and roles are reversed.

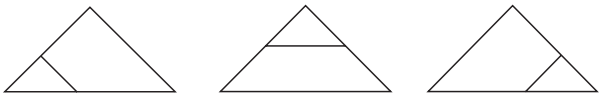
- Which pieces have congruent shapes within the set of 7 tangram pieces? **[the large triangles and the small triangles]**

Display a large and a small triangle to the class. Ask about their size and shape. Obviously one is larger than the other, but point out that corresponding angles are the same size. Encourage students to test this themselves by using their tangram pieces.

Choose a large and a small triangle. Place the small triangle on top of the large with both hypothenuses at the bottom.



Instruct students to leave the larger piece in place, while taking the smaller piece and matching each of its corresponding angles to the larger shape. By comparing the angles of both pieces, students will see that corresponding angles are the same size in similar triangles.



AREA

Use tangram pieces as non-standard units of measure in exploring area of shapes. Permit some free exploration time covering surfaces. Give students a quantity of square cutouts as a unit of measure and ask them to cover the front cover of a book. Have them then count the number of "square units" that were used. As students investigate the concept of area, encourage them to estimate how many units they will need to cover a shape before they actually perform the task.

Draw a master sheet with the following shapes, duplicate, and hand out copies to each pair of students.

$2\frac{3}{4}$" x $2\frac{3}{4}$" square

$2\frac{3}{4}$" x $1\frac{3}{8}$" rectangle

$4\frac{1}{8}$" x $4\frac{1}{8}$" square

$1\frac{3}{8}$" x $1\frac{3}{8}$" square

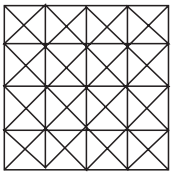
$2\frac{3}{4}$" x $4\frac{1}{8}$" rectangle

Ask each team to find the number of square tangram pieces that will cover each shape. Direct students to record their answers by tracing around the piece each time it is used to cover the shape.

Instruct students to use one of the small triangles as the unit of measure. On a sheet of paper, ask them to trace the various pieces of their tangram set and then determine the area of each using the small triangle. Have then record their answers on a chart:

small triangle	**area = [1 unit]**
square	**area = [2 units]**
medium triangle	**area = [2 units]**
parallelogram	**area = [2 units]**
large triangle	**area = [4 units]**

Ask older students to use the small triangle as the unit of measure to construct a grid like this on a sheet of paper.



Provide copies for younger children.

Use the small triangle to draw a shape on the grid that has an area of 6 units and color it blue.

Expand or add to your shape so that the total area is 9 units. The expansion area should be colored yellow.

Turn your shape into one with an area of $10\frac{1}{2}$ units. Color the additional area red.